§§2A/B Fractions and The Use of Units

Fraction Review:

Fractions are simply another way to write division. A fraction is written $\frac{a}{b}$, where a and b are numbers. Note that a fraction is a number.

Here are some examples:

Ex: $\frac{2}{3}$ is a fraction

Ex: $\frac{\pi}{9}$ is a fraction

Ex: $-\frac{23.1175}{2.35}$ is a fraction.

We can add, subtract, divide, and multiply fractions. We can also simplify fractions.

SIMPLIFICATION OF FRACTIONS

Ex: $\frac{2}{6} = \frac{1}{3}\cdot\frac{2}{2} = \frac{1}{3}\cdot 1 = \frac{1}{3}$

Ex: $\frac{2\pi}{5\pi} = \frac{2\cancel{\pi}}{5\cancel{\pi}} = \frac{2}{5}\cdot 1 = \frac{2}{5}$

Ex: $-\frac{7}{14} = -\frac{1}{2}\cdot\frac{7}{7} = -\frac{1}{2}\cdot 1 = -\frac{1}{2}$

WARNING! The following are all WRONG! NEVER DO THESE UNLESS YOU WANT THE WRONG ANSWER!

Ex: $\frac{2+3}{2} \neq \frac{\cancel{2}+3}{\cancel{2}} \neq \frac{1+3}{1} \neq 4$

Ex: $\frac{2}{7}+\frac{7}{2} \neq \frac{\cancel{2}}{\cancel{7}}+\frac{\cancel{7}}{\cancel{2}} \neq 1+1 \neq 2$

Rule: Never cancel across addition or subtraction!

ADDING AND SUBTRACTING FRACTIONS

You must have common denominators to add or subtract fractions.

Ex: $-\frac{2}{7} + \frac{8}{7} = \frac{6}{7}$

Ex: $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$

Ex: $\frac{7}{3} - \frac{2}{3} = \frac{5}{3}$

Ex: $-\frac{7}{14} + \frac{7}{14} = 0$

Find the common denominator and add/subtract.

Ex: $\frac{2}{3} + \frac{9}{6} = \frac{2}{3}\left(\frac{2}{2}\right) + \frac{9}{6} = \frac{4}{6} + \frac{9}{6} = \frac{13}{6}$

Remember: $\frac{2}{3} + \frac{9}{6} \neq \frac{\cancel{2}}{3} + \frac{9}{\cancel{6}} \neq \frac{1}{3} + \frac{9}{3} \neq \frac{10}{3}$

NEVER CANCEL ACROSS ADDITION OR SUBTRACTION.

Ex: $\frac{3}{8} - \frac{2}{24} = \frac{3}{8}\left(\frac{3}{3}\right) - \frac{2}{24} = \frac{9}{24} - \frac{2}{24} = \frac{7}{24}$

MULTIPLYING FRACTIONS

You DON'T NEED common denominators to multiply.

Ex: $\frac{1}{2} \times \frac{4}{3} = 1 \times \frac{2}{3} = \frac{2}{3}$

NOTE: You can cancel across multiplication.

Ex: $\frac{2}{9} \times -\frac{4}{5} = -\frac{8}{45}$

Ex: $\frac{2}{7} \times \frac{100}{3} = \frac{200}{21}$

DIVIDING FRACTIONS

Dividing fractions is easy. "Flip" the second fraction and multiply.

Ex: $\frac{1}{2} \div \frac{4}{9} = \frac{1}{2} \cdot \frac{9}{4} = \frac{9}{8}$

Definition: The units of a number describe what is being measured or counted.

Examples of Units:

Names of objects:

Apples 8 apples

computers

Cars

people

Length:

feet 5,280 ft

meters

yards

Time:

Seconds 35 sec

Weeks

Weight:

pounds 4.2 lbs

As shown above, units are simply tacked on the number it describes.

Note: While we can easily add and subtract numbers without units, we must be more careful when units are involved. YOU CANNOT ADD OR SUBTRACT numbers if their units do not agree.

Ex: 25 - 5 = 20 This is legal. There are no units involved.

Ex: 25 sec - 5 sec = 20 sec
This is legal since both numbers being subtracted have the same units. Note the answer has the same units.

Ex: 25 mi - 5 sec = ?
This subtraction is not possible. The units do not agree.

Ex: 20 apples + 10 people = ?
This addition is not possible. The units do not agree.

Ex: 20 sec + 10 = ?
This is not possible either since one number has no units.

Multiplication of Units

Numbers with units can be multiplied. In multiplication, units don't have to match.

Ex: 20 sec × 3 = 60 sec

Ex: 5 mi × 2 sec = 10 mi·sec

Note that $mi \cdot sec$ means simply miles times seconds.

Ex: $10\ ft \cdot 10\ ft = 100\ ft \cdot ft = 100\ ft^2$

Note that "ft^2" is read "square feet"

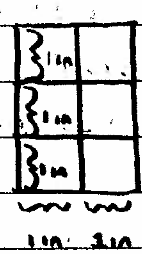
- $mi^2 \rightarrow$ square miles
- $sec^2 \Rightarrow$ square seconds

Ex: $5\ in \cdot 5\ in \cdot 2\ in = 50\ in \cdot in \cdot in = 50\ in^3$

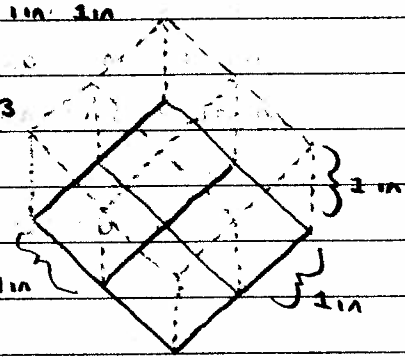
Note that "in^3" is read "cubic inches"

Visually, multiplication of units looks like:

$3\ in \times 2\ in = 6\ in^2$



$2\ in \times 2\ in \times 1\ in = 4\ in^3$



Division of Units

Numbers with units can be divided. The units need not match.

Ex: $5\,mi \div 2\,hrs = 5\,mi \cdot \frac{1}{2\,hrs} = \frac{5\,mi}{2\,hrs}$

Notice we used the rule of fraction division.

Ex: $3\,sec \div 3 = \frac{\cancel{3}\,sec}{\cancel{3}} = 1\,sec$

Note there is no 'sec' in the denominator so it doesn't cancel like the 3s did.

Ex: $100\,m \div 5\,m = \frac{\cancel{100}^{20}\,\cancel{m}}{\cancel{5}_{1}\,\cancel{m}} = 20$

Notice the meters cancel in this example.

Reading Units:

Many unit conversion problems we work with are presented as word problems. There are a few "signal words" you need to know.

Multiplication: a hyphen is sometimes used to represent unit multiplication.

Ex: foot · pounds = foot-pounds.

Note this does not mean foot minus pounds.

raising to second power: "square"

Ex: $mi \times mi = mi^2$ = square miles

raising to the third power: "cubic" or "cube"

Ex: $ft \times ft \times ft = ft^3$ = cubic feet

Division is "per"

Ex: miles per hour = $\frac{mi}{hr}$ = mi ÷ hr

Unit Conversion Factors:

Multiplication by 1 does not change anything. IE:

12 inches · 1 = 12 inches

A conversion factor is a fraction involving two units such that the fraction is equal to 1. For example, there are 12 inches in a foot, so

$$\frac{12 \text{ in}}{\text{ft}} = 1$$

since 12 inches = 1 foot. For the same reason, $\frac{1 \text{ ft}}{12 \text{ in}} = 1$

Unit conversion involving these factors involve fraction operations.

Page 101/2, Table 2.2/2.3/2.4 have some important conversion factors.

Ex: Convert 24 inches to feet:

$$\frac{\overset{2}{\cancel{24}}\ \cancel{\text{in}}}{} \Bigg| \frac{1 \text{ ft}}{\underset{1}{\cancel{12}}\ \cancel{\text{in}}} = 2 \text{ ft}$$

Ex: How many minutes are there in one day?

$$\frac{1\ \cancel{\text{day}}}{} \Bigg| \frac{24\ \cancel{\text{hrs}}}{\cancel{\text{day}}} \Bigg| \frac{60 \text{ min}}{1\ \cancel{\text{hrs}}} = 1440 \text{ min}$$

Ex: How many square feet are in a square yard?

We know:

$$1\text{ yd} = 3\text{ ft}$$

So, square both sides to answer the question

$$1\text{ yd}^2 = 3^2\text{ft}^2 = 9\text{ft}^2$$

So, 1 square yard has 9 square feet.

Ex: How many cubic inches are there in one foot?

We know:

$$1\text{ ft} = 12\text{ in}$$

So, cube both sides:

$$1\text{ ft}^3 = 12^3\text{ in}^3 = 1728\text{ in}^3$$

This says that the conversion factor from cubic feet to cubic inches is

$$\frac{1728\text{ in}^3}{1\text{ ft}^3}$$

Ex: Find the area in square feet of a room with length = 10 ft and a width of 9 ft.

$$\text{Area} = l \cdot w = 10\text{ ft} \cdot 9\text{ ft} = 90\text{ ft}^2$$

Convert this to square yards.

$$\frac{\overset{10}{\cancel{90}}\ \cancel{\text{ft}^2}}{} \cdot \frac{1\text{ yd}^2}{\cancel{9}\ \cancel{\text{ft}^2}} = 10\text{ yd}^2$$

The US Customary System

These are presented in tables 2.2 - 2.4 on pp. 101-2.

The International Metric System

The base units in this system are:

- second → time 's'
- liter → volume 'L'
- meter → length 'm'
- Kilogram → mass 'kg'

Table 2.5 provides the metric "prefixes."

Temperature Conversions

There are three important temperature scales, Fahrenheit, Celsius, and Kelvin. °F is the best-known to Americans. The Kelvin and Celsius scales are identical except for their zero point.

Conversion formulas:

°F → °C : $C = \frac{F - 32}{1.8}$

°C → °F : $F = 1.8C + 32$

°C → K : $K = C + 273.15$

K → °C : $C = K - 273.15$

Ex: Convert 25°C to °F.

Solution: We need °C → °F. Since

$°F = 1.8C + 32$,

$°F = 1.8(25) + 32$

$°F = 45 + 32 = 77\,°F$

Ex: Convert 70°C to K.

Solution: We need °C → K. Since

$K = C + 273.15$,

$K = 70 + 273.15$

$= 343.15\ K$

Ex: Convert 212°F to °C

Solution. We need °F → °C.

$$°C = \frac{F - 32}{1.8}$$

$$= \frac{212 - 32}{1.8}$$

$$= \frac{180}{1.8} = 100\,°C$$

Unit Conversion Example Problems:

Ex: Convert 10 feet to units of inches.

$$\frac{10\ \cancel{ft}}{} \,\Big|\, \frac{12\ in}{1\ \cancel{ft}} = 120\ in$$

Ex How many seconds are in a week?

$$\frac{1\ \cancel{week}}{} \,\Big|\, \frac{7\ \cancel{days}}{1\ \cancel{week}} \,\Big|\, \frac{24\ \cancel{hrs}}{1\ \cancel{day}} \,\Big|\, \frac{60\ \cancel{min}}{1\ \cancel{hr}} \,\Big|\, \frac{60\ sec}{1\ \cancel{min}}$$

$$= 604,800\ sec$$

Ex: Convert 35 gallons to quarts.

$$\frac{35\ \cancel{gal}}{} \Big| \frac{4\ qts}{1\ \cancel{gal}} = 140\ qts$$

Ex: Assuming an exchange rate of

1.5 USD to 1 €

1 USD to 90 ¥

Convert \$1,500 to each of the above.

USD → €

$$\frac{1{,}500\ \cancel{USD}}{} \Big| \frac{1\ €}{1.5\ \cancel{USD}} = 1000\ €$$

USD → ¥

$$\frac{1500\ \cancel{USD}}{} \Big| \frac{90\ ¥}{1\ \cancel{USD}} = 135{,}000\ ¥$$

Convert 3,500 ¥ to €

¥ → USD → €

$$\frac{3{,}500\ ¥}{} \Big| \frac{1\ USD}{90\ ¥} \Big| \frac{1\ €}{1.5\ USD} = 25.9\ €$$

Ex: Find a conversion factor between ft^2 and in^2.

Solution: $1\ ft = 12\ in$

$1^2 ft^2 = 12^2 in^2$

$1\ ft^2 = 144\ in^2$

so, $1 = \frac{144\ in^2}{ft^2}$ and $1 = \frac{1\ ft^2}{144\ in^2}$

Ex: A can has a circular base with area 10 cm^2. If the can is 5 dm high, find its volume in cubic centimeters.

Solution: First note

$$\begin{array}{c|c|c} 5\text{ dm} & 1\text{ m} & 100\text{ cm} \\ \hline & 10\text{ dm} & 1\text{ m} \end{array} = 50\text{ cm}$$

So the can is 50 cm tall. The

VOLUME = BASE AREA x HEIGHT

$= 10\text{ cm}^2 \times 50\text{ cm} = 500\text{ cm}^3$

Ex: A building is 40 yards long and 25 yards wide. It is piled to a height of 3 yards with boxes. Find the area of the building's floor and the volume of the boxes.

Solution:

Floor Area $= 40\text{ yds} \times 25\text{ yds} = 1000\text{ yd}^2$

Volume $= 40\text{ yds} \times 25\text{ yds} \times 3\text{ yds} = 3{,}000\text{ yd}^3$

Ex: Convert 150 yd^2 to square feet.

Solution:

$1\text{ yd} = 3\text{ ft} \Rightarrow 1\text{ yd}^2 = 9\text{ ft}^2$

Thus, our conversion factor is $\frac{9\text{ ft}^2}{1\text{ yd}^2} = 1$

$$\begin{array}{c|c} 150\text{ yd}^2 & 9\text{ ft}^2 \\ \hline & \text{yd}^2 \end{array} = 1{,}350\text{ ft}^2$$

Ex: Convert $3\,\frac{\text{km}}{\text{hr}}$ to meters per minute.

$$\begin{array}{c|c|c} 3\text{ km} & 1000\text{ m} & 1\text{ hr} \\ \hline \text{hr} & 1\text{ km} & 60\text{ min} \end{array} = 50\,\frac{\text{m}}{\text{min}}$$

Review of Powers of Ten.

$10^3 = 10 \cdot 10 \cdot 10 = 1000$

$10^5 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000$

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = .01$

$10^{-5} = \frac{1}{10^5} = \frac{1}{100{,}000} = .00001$

$10^0 = 1$

Multiplying powers of 10: "ADD EXPONENTS"

Ex: $10^0 \cdot 10^7 = 1 \cdot 10^7 = 10^7$

Ex: $10^2 \cdot 10^3 = 10^{2+3} = 10^5$

Ex: $10^5 \cdot 10^{-3} = 10^{5+(-3)} = 10^2$

Ex: $10^1 \cdot 10^3 = 10^{1+3} = 10^4$

Dividing powers of 10: "SUBTRACT EXPONENTS"

Ex: $10^{15} \div 10^5 = \frac{10^{15}}{10^5} = 10^{15-5} = 10^{10}$

Ex: $10^3 \div 10^2 = \frac{10^3}{10^2} = 10^{2-2} = 10^1 = 10$

Ex: $10^7 \div 10^0 = \frac{10^7}{10^0} = 10^7$

Ex: $10^{-10} \div 10^{-15} = \frac{10^{-10}}{10^{-15}} = 10^{-10+15} = 10^5$

Adding and Subtracting Powers of 10:

There are no shortcuts here. Write the numbers in longhand and add or subtract.

Ex: $10^3 - 10^2 = 1000 - 100 = 900$

Ex: $10^4 + 10^5 = 10{,}000 + 100{,}000 = 110{,}000$

Ex: $10^2 + 10^{-2} = 100 + 10^{-2} = 100 + .01$

$= 100.01$

Quiz 1 Topics:

* Fallacies (§1A)
* Logical Operators (§1B).
* Truth Tables (§1B)
* Propositions
* Conditional, converse, inverse, contrapositive
* Sets of numbers, recognizing irrational and rational numbers, and using set notation

§3A Percentages

Review of Percentages:

Per cent means "per 100" or "out of 100."
Per cent is often written as %.

Ex: $50\% = \frac{50}{100} = \frac{1}{2} = .5$

Ex: $25\% = \frac{25}{100} = \frac{1}{4} = .25$

Ex: $157\% = \frac{157}{100} = 1.57$

Ex: $.05\% = \frac{.05}{100} = \frac{5}{10000} = .0005$

Ex: $100\% = \frac{100}{100} = 1$

Convert decimals to percent: Move decimal to the right two places.

Ex: $.15 = 15\%$

Convert fraction to percent: Convert fraction to decimal and then convert as above.